Linear Helicopter Trackers Using Attitude Measurements

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This paper investigates the benefits of using Euler angle and rotor angle information in addition to position information in tracking maneuvering helicopters. The tracking problem as addressed here involves estimating present position and predicting the future position of a highly maneuvering target helicopter. The results of a theoretical error variance analysis indicate that trackers using helicopter mathematical models that include translational, fuselage-attitude, and rotor-angle dynamics have lower position, velocity, and acceleration error variances than conventional trackers. Frequency responses of trajectory prediction errors display lower prediction errors for the trackers using attitude and rotor angle information as compared to conventional trackers. Furthermore, simulations have shown that a tracker using attitude and rotor-angle information would null trajectory prediction errors (following a helicopter maneuver) substantially faster than conventional trackers. A large remaining problem for target trackers is the inability to predict accurately future trajectories when the aircraft is undergoing jinking maneuvers. Nonetheless, the use of attitude information in helicopter trackers helps reduce these trajectory prediction errors.

Introduction

AN aircraft tracker consists of a state estimator (e.g., a Kalman filter) that uses imperfect measurements of a target's motion to calculate estimates of present target motion and a predictor to calculate future motion of the target. In a typical battlefield environment, a target tracker may have to accurately predict the future position of an unfriendly aircraft up to 5 s into the future. Conventional target trackers use radar to determine target position and Doppler radar to determine range rate. Since modern helicopters are increasingly able to perform more highly accelerated "jinking" maneuvers, these conventional trackers with no direct knowledge of target acceleration have difficulty in predicting future helicopter positions.

The purpose of this paper is to investigate the benefits to helicopter trackers of modeling and measuring the orientation of the helicopter's fuselage and the orientation of the helicopter's rotor system. Since orientation is strongly related to acceleration, knowledge of orientation helps estimate target acceleration and therefore improves the accuracy of trajectory predictions. Three different approaches are discussed in this paper for quantifying these benefits. The three approaches include a theoretical error variance analysis, a frequency response analysis of trajectory prediction errors, and a transient response analysis on simulated helicopter trajectory data.

The tracking problem addressed in this paper is a much simplified version of the military fire control problem. We assume that the problem is linear and therefore employ simple and intuitive linear analysis techniques. We formulate a scenario in which we can assume that the target is at approximately constant range from the tracker therefore allowing us to greatly simplify the lead-angle prediction problem. We make these simplifications to help expose the importance of using attitude measurements in tracking without undue mathematical complexities. It should be recognized that these simplifications are not necessarily desirable in a realistic fire control system.

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Figure 1 shows the scenario we have analyzed. The helicopter is flying low to the ground in a northerly (x) direction at approximately 95 ft/s. The stationary tracker is about 6600 ft to the west. In this 17-s scenario, the helicopter range varies from 6611 to 6562 ft giving a typical projectile time of flight of 2–2.015 s. For simplicity we have assumed a constant time-of-flight of 2 s in our analyses. It should be recognized that times of flight from 0 to 5 s are just as likely.

We consider helicopter maneuvers in only the vertical (x-z) plane to further simplify our analysis. For the scenario in Fig. 1, lateral helicopter maneuvers (y direction) are not good pilot strategy since the projectiles would be moving rapidly in this same direction. Thus, lateral motion would not take the helicopter out of harm's way.

The central idea in this research is that in most cases a change in helicopter orientation will precede a change in target trajectory. Therefore, orientation measurements provide valuable lead information useful for trajectory prediction.[1,3-6] For example, if a pilot wishes to translate forward he will rotate (change the attitude of) the rotor system down at the nose of the vehicle. This will produce a forward component of thrust. Following the rotation of the rotor system, the vehicle body will pitch down (another change in attitude) giving an even greater component of forward thrust. Following these rotations. Translational motion will occur when the forward acceleration (proportional to forward thrust) is integrated into forward velocity and integrated again into forward translation. A tracker that "sees" these orientation movements (thereby gaining knowledge of the orientation of the thrust vector) will know beforehand the future translational motion of the helicopter, resulting in more accurate trajectory predictions.

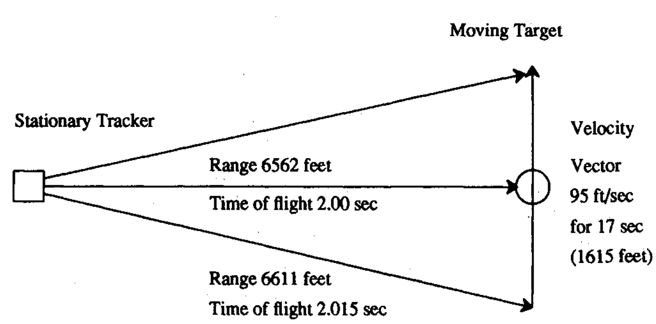

Fig. 1 Tracking scenario.

Conventional trackers do not use orientation information, and they have crude approximations to the vehicle's accelerations. The conventional tracker discussed in this paper is based on an "α-β model."[1] The state equation of the α-β model is

$$\begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w \tag{1}$$

where, for example, x is the inertial x position. Note that w is zero-mean Gaussian white noise. The α-β model assumes that the acceleration $\ddot{x}$ is white noise. We compute the parameters of the tracking filter by developing a Kalman filter for the state equation given in Eq. (1). The α-β models can also be applied to inertial y and z directions.

The prediction models are typically based on the Taylor series expansion of the predicted position about estimates of the present position, velocity, and acceleration from the tracking filter. For example, for the predicted x position state, where T is the prediction time:

$$x_P(t) = \hat{x}(t+T) = \hat{x}(t) + T\dot{\hat{x}}(t) + \tfrac{1}{2}T^2\ddot{\hat{x}}(t) + \cdots \tag{2}$$

The optimal predictor based on the α-β model assumes that the acceleration is zero for its prediction model and uses only the first two terms in this equation. Because of this, for highly accelerated maneuvers prediction errors for the α-β model are large. For the new trackers studied in this paper, however, accelerations are modeled as functions of the orientation of the fuselage and rotor system leading to more accurate estimates of acceleration and increased reliance on the third term in Eq. (2).

Remote Determination of Helicopter Attitude

Trackers studied in this paper use knowledge of the orientation of the fuselage of the target helicopter as well as the orientation of the rotor system (tip-path plane) with respect to the fuselage. It is not the intent of this paper to discuss in detail methods for obtaining this data from an unfriendly helicopter.

Recent research[2] has shown that video imagery from a remote camera viewing a fixed-wing aircraft can be used to determine the orientation of the aircraft with respect to an inertial frame (e.g., Euler angles). It is assumed that a technique[2,13] can be found to determine the orientation of the helicopter fuselage. It is further assumed that the orientation of the rotor system can also be determined by processing video imagery, although the accuracy of rotor attitude might be less than for fuselage attitude. A fast frame rate may be required to capture images of the rotor itself.

Linear Modeling of Helicopters

Accurate mathematical modeling of helicopter dynamics is a complex endeavor. Reference 9 should be consulted for a careful study of helicopter modeling. Though complicated nonlinear mathematical models frequently bring increased modeling accuracy, the complexity of the model can sometimes obscure the important or essential terms within the model. Reliance on nonlinear dynamic models eliminates the researcher's ability to use many linear system analysis tools (such as frequency response methods), which are useful in analyzing dynamic systems. Furthermore, although improved modeling accuracy is usually desirable, there is a point of diminishing returns in the tracking application. This is because the tracker is ignorant of the sizable forces and moments caused by the pilot's commands to the vehicle's control system, and in many cases this error source can be much larger than other errors (such as those due to linearization). For the aforementioned reasons, several simplified linear dynamic models of helicopters were developed in this paper and used for target tracking. It is recognized that in the implementation of a practical target tracker, several important nonlinearities will need to be reintroduced.

Excluding gravity, the primary contribution to a helicopter's acceleration is due to the thrust force generated by the rotor blades. To a first approximation, the thrust vector can be modeled as being perpendicular to a plane formed by the tips of the rotor blades as they travel around the rotor hub. This plane is called the tip-path plane and is oriented with respect to the fuselage by the longitudinal flapping angle a_1 and the lateral flapping angle b_1. The magnitude of the thrust vector is approximately proportional to the coning angle a_0 of the rotor system.[9] Figure 2 shows the tip-path plane and these rotor angles.

A nonlinear mathematical model of the helicopter has been developed and described in Ref. 11. When linearized about a steady level flight path (with all reference angles zero) the following equations result:

$$\ddot{x} = -g(\theta + a_1) + \text{white noise} \tag{3}$$

$$\ddot{y} = g(\phi + b_1) + \text{white noise} \tag{4}$$

$$\ddot{z} = -K_z(a_0 + c) + \text{white noise} \tag{5}$$

$$\dot{c} = \text{white noise} \tag{6}$$

$$\ddot{\phi} = K_\phi b_1 + \text{white noise} \tag{7}$$

$$\ddot{\theta} = K_\theta a_1 + \text{white noise} \tag{8}$$

$$\ddot{a}_0 = \text{white noise} \tag{9}$$

$$\ddot{a}_1 = \text{white noise} \tag{10}$$

$$\ddot{b}_1 = \text{white noise} \tag{11}$$

These equations are called the "17th order helicopter model." Note that x, y, and z are aircraft position coordinates in an inertial frame; ψ, θ, and ϕ are fuselage Euler angles; and a_0, a_1, and b_1 are rotor-system angles as defined in Fig. 2.

Equations (3–11) result from the following assumptions:

1) The magnitude of the thrust vector is proportional to coning angle a_0 and a bias term c. In steady flight vertical acceleration, $\ddot{z}$ is zero when $a_0 + c = 0$.

2) The direction of the thrust system is perpendicular to the tip-path plane. The tip-path plane is oriented with respect to the fuselage by tip-path plane angles a_1 and b_1.

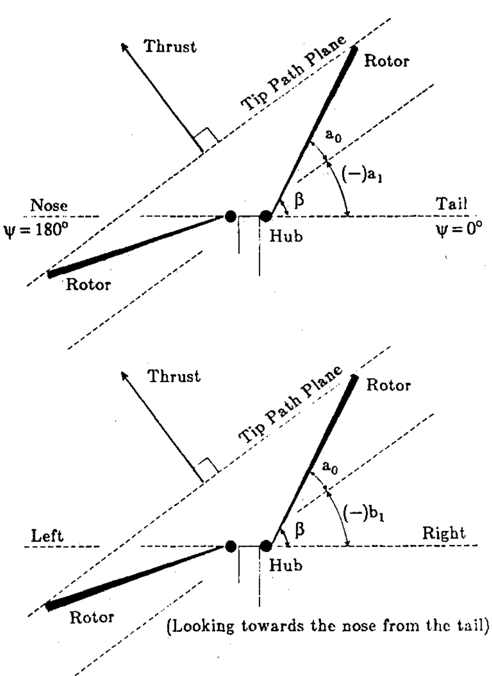

Fig. 2 Helicopter rotor angles and the tip-path plane.

3) The magnitude of rotor thrust approximately balances the vehicle's weight.

4) Tilting the thrust vector produces a pitching moment proportional to a_1 and a rolling moment proportional to b_1.

5) Gaussian statistically independent white noise is used to represent uncertainty in these linear differential equations.

6) Acceleration of gravity is g.

One should note that this model requires only three vehicle-specific constants (K_z, K_ϕ, and K_θ). It is assumed that the optical image processor can determine the helicopter type (as well as the fuselage and tip-path plane attitude) and that the vehicle-specific constants are known for each aircraft type or can be approximated with sufficient accuracy. Note also that the mathematical models for rotor dynamics are very naive. This is done to minimize complexity and the number of vehicle specific constants. Other studies[11] have shown that improved results are possible if the rotor angle dynamics (equations for a_1 and b_1) are modeled as underdamped, second-order systems.

An even simpler model is possible if one ignores lateral translational motion, lateral flapping angle b_1, and the bias in coning angle c. The following 10th-order "simplified helicopter model" results.

$$\ddot{x} = -g(\theta + a_1) \tag{12}$$

$$\ddot{z} = -K_z a_0 \tag{13}$$

$$\ddot{\theta} = (Wd/I_{yy})a_1 \tag{14}$$

$$\ddot{a}_0 = \text{white noise} \tag{15}$$

$$\ddot{a}_1 = \text{white noise} \tag{16}$$

Note that θ is the Euler pitch angle, W is the weight of the vehicle, and I_{yy} is the mass moment of inertia about the y axis. Typical values for the helicopter parameters were chosen for this model.[9] The proportionality constant in the z acceleration equation was chosen to be $K_z = 500$. The values for the other parameters were chosen to be $d = 7.5$ ft, $W = 15{,}000$ lb, and $I_{yy} = 40{,}000$ slug-ft^2.

More simplifications are possible if one ignores the longitudinal flapping angle a_1. Ignoring the flapping angles is justifiable in many cases where the Euler angles are large compared to the flapping angles, as is the case in violent maneuvers. The resulting equations of motion, called the "α-δ model," are

$$\ddot{x} = -g\theta \tag{17}$$

$$\ddot{z} = -K_z a_0 \tag{18}$$

$$\ddot{\theta} = \text{white noise} \tag{19}$$

$$\ddot{a}_0 = \text{white noise} \tag{20}$$

In Eqs. (17) and (18) one can see the direct and simple relationship between acceleration and orientation. The mathematical models for orientation, Eqs. (19) and (20), are very naive, but no more so than the equation for acceleration in the α-β model of Eq. (1).

Throughout this paper we study tracking filters based on the various helicopter models already described. A tracking filter consists of a steady-state Kalman filter-state estimator based on a particular state equation model. We name the tracking filter with the same name as the model upon which the Kalman filter is based.

Tracker Performance Evaluation

This section describes the performance of target trackers that use attitude measurements to help track maneuvering helicopters. Three different approaches are taken so that different aspects of the tracker can be studied using different assumptions. This gives a broader knowledge base upon which to evaluate and understand the trackers.

The three approaches are described in the following three subsections. The first method, "theoretical steady state error variance analysis," can be thought of as steady-state statistical analysis of the tracker giving an indication of long-term average behavior of the tracker when the helicopter is driven by a random process. The second method, "prediction error frequency response," concentrates on the trajectory prediction problem in a noise-free environment under the assumption that the pilot is forcing the helicopter to undergo sinusoidal motion. The third method, "tracking a simulated helicopter trajectory," is perhaps the most revealing because the dynamic behavior of the helicopter is quite realistic, being neither random nor sinusoidal.

Theoretical Steady-State Error Variance Analysis

To determine the relative value of the different mathematical helicopter models developed in the preceding section, a theoretical steady-state error variance analysis was conducted. The approach described in Ref. 17 was used to minimize the state estimation errors between an "actual" state equation model and estimates from a Kalman filter based on a different lower-order state equation model. The Kalman filter in this case was suboptimal because it was based on the wrong model. The "actual system" was of higher order and contained more sophisticated modeling of the dynamics of a real aircraft in flight than the mathematical models used by the aircraft trackers.

The actual system was defined as

$$\dot{x} = Ax + Bw \tag{21a}$$

$$z = Hx + v \tag{21b}$$

where w was zero-mean Gaussian white process noise, and v was zero-mean Gaussian white measurement noise.

The state model on which the tracker Kalman filter was based was called the "filter system" and was defined as

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}\bar{w} \tag{22a}$$

$$z = \bar{H}\bar{x} + \bar{v} \tag{22b}$$

where $\bar{w}$ was zero-mean Gaussian white process noise and $\bar{v}$ was zero-mean Gaussian white measurement noise. The overbars were used to indicate that the filter system was different from the actual system. However, it was defined that the measurements of the filter system were the same quantities as for the actual system. The commonly employed statistical assumption involving $\bar{x}(0)$, $\bar{w}$, and $\bar{v}$ were used in this work.

The linear Kalman filter based on the filter system was

$$\dot{\hat{x}} = (\bar{A} - \bar{K}\bar{H})\hat{x} + \bar{K}z \tag{23}$$

where $\bar{K}$ was the Kalman gain matrix. We assumed that this Kalman filter was allowed to reach steady state so that the gains $\bar{K}$ were constant.

The Kalman filter is considered an "optimal" state estimator in the sense that it will produce best estimates of $\bar{x}(t)$ given measurement data up to time t such that the error variance $E\{[\bar{x}(t) - \hat{x}(t)][\bar{x}(t) - \hat{x}(t)]^T\}$ is minimized. The conditions of optimality are based on the assumptions that the dynamics of the system the filter is tracking are known exactly and used as the system model for the filter. However, the Kalman filters in this analysis were suboptimal because the actual systems and the filter systems were not the same.

Following the approach of Ref. 7, an error vector was defined as

$$\tilde{x} = Cx - \hat{x} \tag{24}$$

where C is a transformation matrix that reduces the actual state vector to the lower order of the Kalman filter estimate vector.

A variance matrix X is defined as

$$X = E\left[\begin{Bmatrix} x \\ \tilde{x} \end{Bmatrix}(x^T, \tilde{x}^T)\right] = \begin{bmatrix} E(xx^T) & E(x\tilde{x}^T) \\ E(\tilde{x}x^T) & E(\tilde{x}\tilde{x}^T) \end{bmatrix} \tag{25}$$

The steady-state solution of this matrix was found by solving a steady-state Lyapunov equation developed by Ref. 7.

Cost functions to be minimized were then developed. These cost functions consisted of weighted sums of elements in the variance matrix. The weighting factors were derived by squaring each term in the second-order Taylor series expansion of the prediction error equation. For example, the cost function for the inertial x position was

$$J_x = E(\tilde{x}^2) + 4E(\dot{\tilde{x}}^2) + 4E(\ddot{\tilde{x}}^2) \tag{26}$$

where the prediction time T was defined as a constant 2 s. In Eq. (26), $\tilde{x}$, a scalar, denotes the error between the actual x inertial position and estimated x inertial position. Although minimizing this cost function would not minimize prediction errors, the weighting placed importance on minimizing the velocity and acceleration error variances, which were important terms when attempting to reduce prediction errors.

Since the Kalman filters were suboptimal, they had to be "tuned" to obtain the "best" Kalman gains that would result in lowest values of the cost functions. The Kalman gains were a function of the process white noise intensities and the measurement noise intensities. The measurement noise intensities were assumed functions of the equipment used to gather the trajectory information. Hence, these statistics were held constant and the tuning procedure consisted of varying the intensities of the process noise until the lowest values for the cost functions were recorded. The standard deviations of the discrete measurement white noise were 20 ft for the position measurements, 20 ft/s for the position rate measurements and 2 deg for all angle measurements.

The helicopter model used as the actual system is presented in Fig. 3. This model is called the simplified helicopter model as discussed earlier [Eqs. (12–16)]. Three different models were used as filter systems. These included simple α-β models in x and z directions, the α-δ model [Eqs. (17–20)], and the simplified helicopter model itself. The latter model was the optimal model since it was the same as the actual system.

Results of this theoretical analysis are presented in Tables 1 and 2. These tables present the position, velocity, and acceleration root mean square (rms) errors for the x position state when the cost functions [Eq. (26)] were minimized. The rms errors are the square root of the error variances. Note that the cost functions for the α-β model used only the terms involving the position and velocity error variances. All data presented in Tables 1 and 2 are the result of the optimization procedure just described. The cost function [Eq. (26)] was minimized by adjusting the process noise intensity in the filter system.

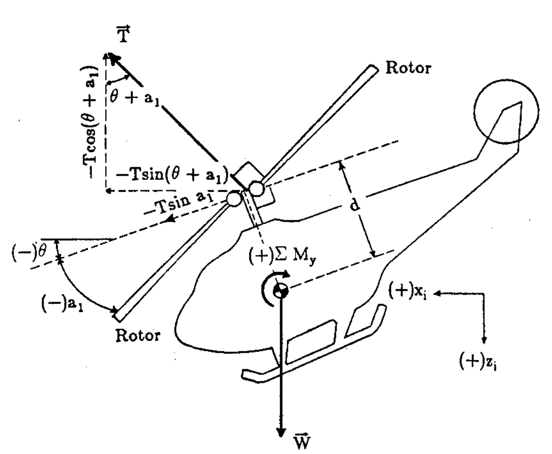

Fig. 3 Simplified helicopter model.

Table 1 shows that the trackers that used Euler and rotor angle information had lower rms errors. In comparing the performances between the simplified helicopter model and the α-δ model, it could be seen that there were 79, 97, and 97% reductions in position, velocity, and acceleration rms errors for the simplified helicopter model. However the α-δ model showed improved performance over the α-β model. There were 65 and 96% reductions in the position and velocity rms errors for the α-δ model.

Table 2 shows that lower rms errors were seen as the number of measurements available to the Kalman filter was increased. There were 59, 92, and 94% reductions in position, velocity, and acceleration rms errors when attitude and rotor angle measurements were used in addition to position and position-rate measurements. There were 78, 97, and 97% reductions in these errors when attitude and rotor angle measurements were used in addition to only position measurements. Reductions in errors due to the use of additional measurements could also be seen for both the α-δ and α-β models.

In comparing the last line of Table 1 with the last line of Table 2, it could be seen that tracking performance improved by just modeling Euler and rotor angles without measuring these angles. Similar results were found for the z direction.[10]

Prediction Error Frequency Response Analysis

The previous section examined errors in the estimation of present vehicle position, velocity, and acceleration. In this section, errors made during the trajectory prediction part of the tracking problem are studied. In the previous section random inputs (process noise) were assumed and output error variances were examined. In this section sinusoidal inputs are assumed and the size of trajectory prediction errors are studied. We examine different input types in order to more fully understand the performance of the filters. No one type of input is best under all circumstances.

This section describes a frequency response analysis of the tracking filters designed in the last section. The constant filter gains used to obtain the results of Table 1 were used in this analysis. It was assumed that the actual system [Eqs. (12–16)] was driven by deterministic sinusoidal inputs in the rotor angle acceleration equations [Eq. (15) or (16)]. Process and measurement noise were assumed to be zero in this completely deterministic analysis.

One measure of the size of the trajectory prediction errors is the magnitude frequency responses of the normalized prediction errors. For the x direction, the normalized prediction

Table 1 Theoretical rms errors in x for various tracker models

Filter system	Measurements	rms errors		
		x, ft	$\dot{x}$, ft/s	$\ddot{x}$, ft/s^2
Simplified helicopter model[a]	x, θ, a_1	1.1	0.33	0.38
α-δ model	x, θ	5.2	10.0	12.0
α-β model	x	15.0	260.0	——

[a]This is also the "actual system."

Table 2 Theoretical rms errors in x for various measurement vectors

Filter system	Measurements	rms errors		
		x, ft	$\dot{x}$, ft/s	$\ddot{x}$, ft/s^2
Simplified helicopter model[a]	x, $\dot{x}$, θ, a_1	1.1	0.33	0.38
	x, θ, a_1	1.1	0.33	0.38
	x, $\dot{x}$	2.7	4.3	6.4
	x	5.0	9.5	12.0

[a]This is also the "actual system."

error is defined as

$$\left| \frac{x_A(j\omega) - x_P(j\omega)}{x_A(j\omega)} \right| \tag{27}$$

where $x_A(t) = x(t+T)$, which is the exact x position state advanced T s into the future. The $x_P(t)$ is the prediction of $x(t)$, T s into the future, based on the filter's state estimates at time t as defined in Eq. (2). Similar definitions hold for the z position state.

Normalized prediction errors are used because the magnitudes of the frequency responses for the actual system and the Kalman filter attenuate as frequency is increased. Therefore, the magnitudes of the unnormalized prediction errors are large at low frequencies and small at high frequencies. This gives the false impression that problems in the prediction scheme occur at the lower frequencies. For example, normalized prediction error magnitudes of 0 dB indicate that the prediction errors are equal in magnitude to the response of the actual system. This is considered very poor performance. Magnitudes of -20 dB indicate the prediction errors are one-tenth of the response of the actual system.

The frequency response of the advanced exact position states were obtained from the following relationship, where, if

$$\frac{x(j\omega)}{u(j\omega)} = G(j\omega), \qquad \text{then} \qquad \frac{x_A(j\omega)}{u(j\omega)} = G(j\omega)e^{j\omega T} \tag{28}$$

Note that the transfer function G was obtained from the actual system (the simplified helicopter model). A simlar relationship held for the z position state.

The frequency responses of the predictions were obtained by solving for the transfer functions between the sinusoidal inputs to the actual system and the state estimates from the Kalman filter. For example, the x prediction frequency response for the filter system of the simplified helicopter model was obtained from the following transfer function,

$$\frac{x_P(s)}{u(s)} = \frac{\hat{x}(s)}{u(s)} + T\frac{\dot{\hat{x}}(s)}{u(s)} - \frac{1}{2}T^2 g\left(\frac{\hat{\theta}(s)}{u(s)} + \frac{\hat{a}_1(s)}{u(s)}\right)$$
$$- \frac{1}{3!}T^3 g\left(\frac{\dot{\hat{\theta}}(s)}{u(s)} + \frac{\dot{\hat{a}}_1(s)}{u(s)}\right) - \frac{1}{4!}T^4 g\frac{Wd}{I_{yy}}\frac{\hat{a}_1(s)}{u(s)} \tag{29}$$

where s is the Laplace variable and $u(s)$ is the sinusoidal input to the acceleration of a_1. The prediction model for the α-δ model was formulated from Eq. (29) by simply removing all terms involving the rotor angle a_1. The prediction model for the α-β model contained only the first two terms in Eq. (29).

Using these relationships, the magnitude frequency response of the relative prediction errors were calculated using a prediction time of 2 s. Figure 3 presents the results of the relative prediction errors for the x position state when the input was $\ddot{a}_1$. The figure labels the maneuver region as being between 0.32 and 1 rad/s. This was considered a typical range for "jinking" maneuvers of helicopters. Within this region, Fig. 4 shows that the trackers that used angle information had, on the average, prediction errors that were between three and six times lower than the prediction errors of the α-β model. Similar results were found for the prediction errors in the z direction when the input was $\ddot{a}_0$.[10]

Several more important points can be made from Fig. 4. Assume that the pilot were to adopt a sinusoidal trajectory as a way to confuse the tracker. Figure 4 shows that the tracker will have significantly greater errors if the frequency of his sinusoidal motion is large (>1 rad/s). On the other hand, a lower frequency sinusoidal trajectory leads to smaller tracker prediction errors. Furthermore, trackers that use attitude information are far superior to the other trackers when the maneuver frequency is <1 rad/s but poorer at frequencies >1 rad/s. Figure 4, therefore, contains information about desired pilot strategy and about comparative tracker performance.

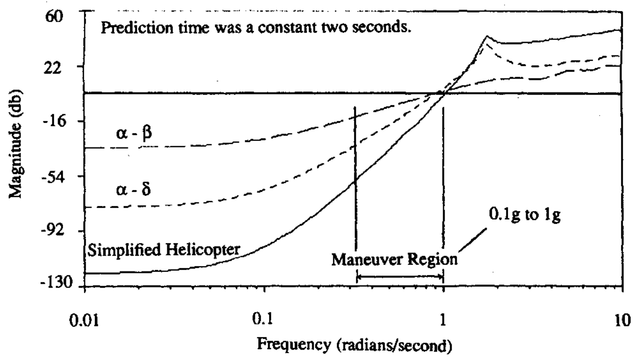

Fig. 4 Magnitude frequency responses of the normalized trajectory prediction errors for x.

Tracking a Simulated Helicopter Trajectory

In this section several trackers were used on a simulated helicopter trajectory in order to examine the transient properties of the trackers in a more realistic manner. A nonlinear simulation program entitled ARMCOP was used to simulate flight data of a generic helicopter. This helicopter was considered to be the actual system in this tracking study. The simulated flight data was obtained from NASA Ames Research Center, and the trajectory was known as a "dolphin" maneuver.[8] The largest accelerations were in the inertial z direction, and the tracking studies concentrated on this direction.

For this analysis the equations of motion for the filter system consisted of the equations described earlier as the 17th order helicopter model. The constants K_z, K_ϕ, and K_θ were vehicle specific parameters, therefore, they would differ from one helicopter to another. For this maneuver their values were found experimentally to be 640, 1, and 1, respectively. If the attitude of the vehicle is determined by an image processor, then that same device may be able to identify the type of helicopter being tracked. That identification might help the tracker determine these vehicle specific parameters.

The maneuver was tracked using a discrete time Kalman filter based upon an α-β model and a 17th order helicopter model. Since three inertial directions were involved, the α-β model consisted of three sets of equations such as Eq. (1), one set for each inertial direction. Simulated noisy measurement data was obtained by adding discrete Gaussian white noise to the "exact data" of this trajectory. The discrete values of the standard deviations of the measurement noise were 10 ft for the position measurements, 10 ft/s for the position rate measurements and 0.5 deg for all angle measurements. The reader is cautioned that these are not the same as those used for the theoretical study state error variance analyses.

The present position estimation performances were similar for both trackers. However, Fig. 5 overplots the exact accelerations and estimated accelerations in the z direction for the tracker using the 17th order model. This figure shows that during the region of highest acceleration, the tracker using the 17th order model made accurate predictions of helicopter acceleration. Recall that the α-β model predicts with zero acceleration.

The prediction model in the z direction for the 17th order tracker was

$$z_P(t) = \hat{z}(t) + T\dot{\hat{z}}(t) - \tfrac{1}{2}T^2 K_z(\hat{a}_0 - \hat{c}) \tag{30}$$

Note that the coning angle was assumed constant in this prediction equation.

Figure 6 compares the prediction performances in the z direction of the α-β and the 17th order models when their tracking filters were tuned to give the lowest possible estimation errors. The large prediction errors associated with the 17th order model in the first few seconds were due to the initial transients in estimating the coning angle's bias state c. The region concentrated on in this study, however, was from 11.5

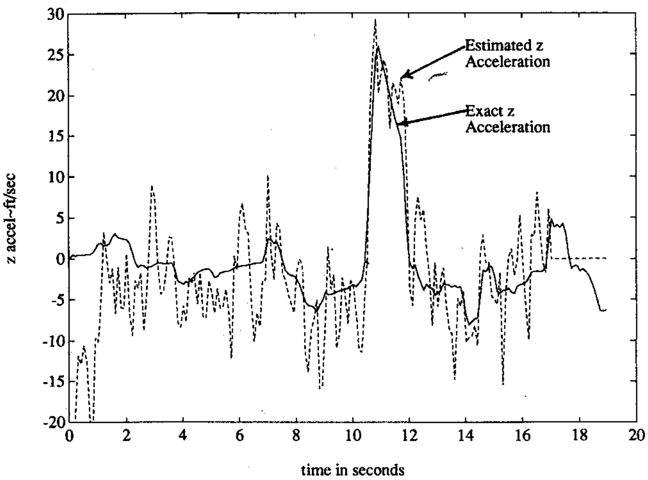

Fig. 5 Exact and 17th order model estimates of the translational accelerations in the inertial z direction.

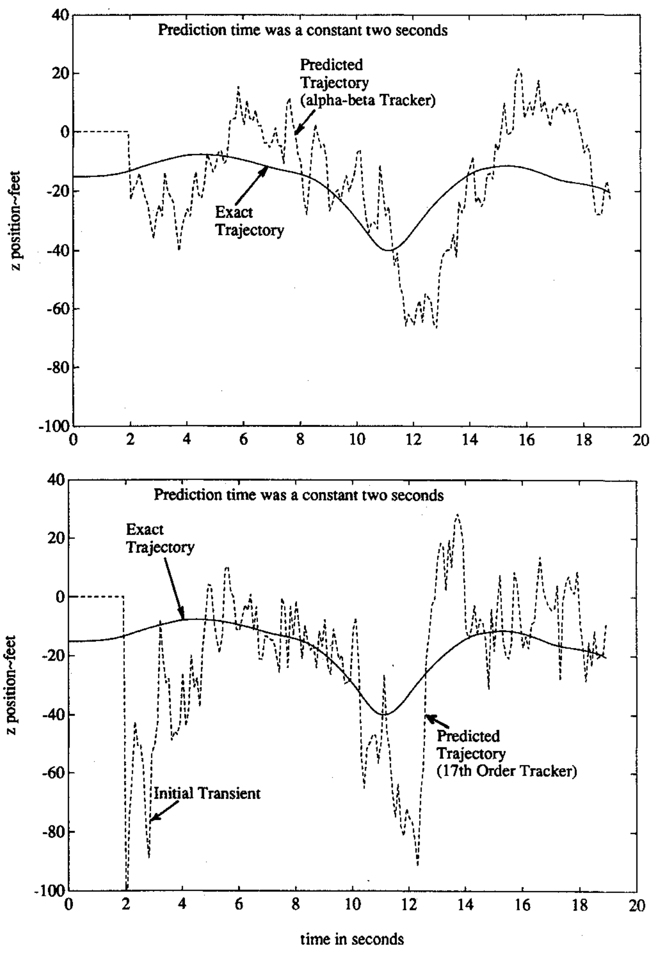

Fig. 6 Exact and predicted trajectories in the inertial z direction for the α-β and 17th order trackers' best runs.

to 13 s. Maximum accelerations in the z direction occurred during this time interval. Since prediction time was a constant 2 s, the predictions were based on state estimates calculated between 9.5 and 11 s.

Notice that the speed of response for the 17th order model was substantially better than that of the α-β model. Although the overshoots in the 17th order model's predictions were larger than those of the α-β model, the tracker recovered from the change in motion in the z direction of the helicopter at

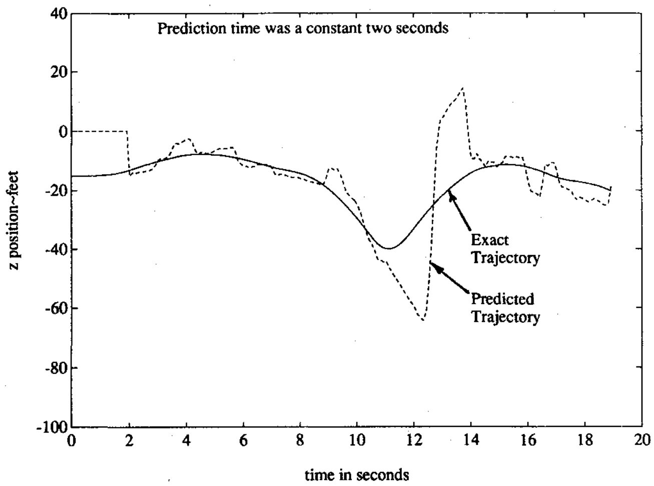

Fig. 7 Predicted trajectory in the inertial z direction using the dolphin maneuver's exact data.

approximately 12.5 s; whereas the α-β model did not recover until approximately 14 s. The 17th order model's predicted trajectory intersected the path of the helicopter's trajectory in this region, while the α-β model was still lagging behind in its predictions of the vehicle's location.

To gain insight into why the predicted trajectory overshoots were so large in Fig. 6, the second-order prediction model using exact values for the position, velocity, and acceleration terms was studied. This model, known as the "exact predictor" because it uses exact values of $z(t)$, $\dot{z}(t)$, and $\ddot{z}(t)$, is

$$z_P(t) = z(t) + T\dot{z}(t) + \tfrac{1}{2}T^2\ddot{z}(t) \tag{31}$$

Figure 7 plots the exact trajectory and the predictions obtained from the exact predictor. The large overshoots of the exact predictor show that a large percentage of the 17th order model's prediction overshoots were due to the prediction scheme and not the estimates from the Kalman filter. The prediction error recovery time for the exact predictor was close to the recovery time of the 17th order model's predictions and shows that prediction performance was greatly limited by the prediction scheme.

Conclusions

This paper describes and analyzes a new class of helicopter target trackers that use measurements of fuselage and tip-path plane angles to supplement radar measurements. This approach allows for more accurate estimates of target acceleration because of the direct relationship between orientation and acceleration. More accurate estimates of vehicle acceleration leads to more accurate predictions of future helicopter motion.

The theoretical work presented in this paper showed that helicopter trackers that used body attitude information had lower position, velocity, and acceleration rms estimation errors than conventional trackers. Trackers that used rotor-angle information in addition to body-attitude information had even lower errors. Frequency responses of prediction-error magnitudes also showed that these new trackers had improved performances over conventional trackers over the frequency range typical of jinking maneuvers.

Transient response analysis of a moderate helicopter maneuver showed that the linear tracker that used attitude and rotor-angle information was able to reduce its prediction errors following rapid changes in the flight path substantially faster than the conventional tracker.

Trackers that use angle information were shown in this paper to be relatively simple and to have few vehicle-specific constants. The results show conclusively that substantial improvements in the performance of helicopter trackers can be gained using attitude measurements together with simple mathematical tracker models for the orientation degrees of freedom.

It was found that a large percentage of the overshoots noted in the predicted trajectories of the dolphin maneuver were the result of the prediction part of the tracker problem and not due to the estimation of the present trajectory. Unfortunately, a tracker does not have knowledge of the vehicle's future jinking behavior so there will always be errors when predicting into the future, even with exact knowledge of the present trajectory. Because of this, prediction error recovery time is an important property of target trackers. This analysis highlighted the fact that errors due to incorrectly predicting the future often dominate this problem.

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